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D-strings and F-strings from string loops

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Abstract

Since the background fields of the string low energy action are supposed to be the long range manifestation of a condensate of strings, the addition of world sheet actions to the low energy effective action needs some string theoretic explanation. In this paper we suggest that this may be understood, as being due to string loop effects. We first present arguments using an equation due to Tseytlin and then more rigorously in the particular case of type IIB theory by invoking the Fischler-Susskind effect. The argument provides further justification for SL$(2, Z)$ duality between D-strings and F(fundamental)-strings. In an appendix we comment on recent attempts to relate the type IIA membrane to the 11-dimensional membrane.

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1 Introduction

There has been astonishing progress in the last year in identifying and elucidating connections between perturbatively different formulations of string theory.[1] Nevertheless, an organizing (dynamical) principle that would enable one to understand this bewildering variety of connections is lacking. In perturbative string theory the relevant dynamical principle is world sheet Becchi-Rouet-Stora-Tyutin (BRST) (or conformal) invariance. This requirement leads to equations of motion for the background fields. However, BRST invariance conditions, being short distance effects on the world sheet, will yield the same background equations independent of topology, so that the classical string background equations will be unmodified. Clearly, this cannot be correct and the resolution was proposed by Fischler and Susskind [2]. They pointed out that conformal anomalies can arise from boundaries of moduli space for genus higher than zero, and they showed that the requirement of BRST invariance for the sum of contributions from the genus zero and higher order terms would modify the classical equations with stringy quantum corrections. In this paper we will argue that this dynamical principle has some relevance to the recent developments.

String theory dualities require that classical solutions carrying Ramond-Ramond (R-R) charges be treated on a par with elementary string states [3]. The former are associated with non-perturbative effects while the latter are the perturbative spectrum of the string which does not have any state carrying R-R charges. Nevertheless, in a recent seminal paper it was shown by Polchinski [4] that the R-R charge carrying classical solutions should be identified with the so-called D-branes, i.e., branes to which open string ends are constrained to move on. Polchinski's paper opened up the possibility of extending the use of world sheet methods to formulate a systematic treatment of non-perturbative effects.

[1] It is hardly possible to list all the important papers here so we just mention one key paper which stimulated a lot of the subsequent work [1].

In this paper we hope to make a small contribution to this program by considering the effects of R-R charge-carrying D-strings in type IIB theory. Now, it is believed that the effect of having the D-brane is to add to the effective 10 dimensional action the $(p+1)$-dimensional action of the D-(p-)brane. This corresponds to the addition of the fundamental string action to the effective action by Dabholkar et al [5], in order to support the singular string solutions of the effective action. Indeed the type IIB string is particularly suited for the study of this correspondence since it is expected to have duality relating NS-NS fields and R-R fields in the effective action [6], [7]. In particular in the work of Schwarz [7] a formula for the string tensions of strings carrying both NS-NS and R-R charges is derived. These strings were later interpreted by Witten [8] as bound states of F(fundamental)-strings and D-strings.

However, there is something strange about the addition of a world sheet action to the low energy effective action of string theory. This is usually justified in analogy with particle actions coupled to external fields. But unlike the case there, in string theory the background fields of the low energy action are not really external fields in which strings propagate—they are expected to be condensates of strings. At resolutions larger than the string scale the stringy nature of the underlying reality will not be manifest. As one approaches the string scale the description in terms of smooth fields (and geometry) should break down. In particular, in regions of high curvature one expects the effective low energy action description to be invalid, and presumably the singularity has no physical significance in string theory. The fact that the string coupling vanishes as one approaches the singularity[2] is perhaps a reflection of this.

What we will seek in this paper is a world sheet justification for adding D-(and F-) string actions to the low energy effective actions. Indeed, the fact that these actions are (one or two) powers of e^{ϕ} down from the tree level effective action suggests that they should arise as loop effects. We will find that the Fischler-Susskind argument provides a rationale correcting the tree-level equations in this way. In particular, we will also

[2]See, for example, the review of Duff et al [9].

support from the world sheet point of view the $N = 2$ supergravity argument for the absence of dilaton couplings of R-R fields. The modified Dirac-Born-Infeld (DBI) action that we get is then shown to be of the same form as that of the fundamental string action with tension given by Schwarz's formula.

The machinery that we use in this paper was developed in a remarkable series of papers by Callan, Lovelace, Nappi, and Yost (CLNY), culminating in [10]. The construction of the D-brane state using their method was first done by M. Li [11]. During the course of this investigation, several papers ([12], [13], [14]) appeared that have some overlap with our work (particularly the last paper). We feel, however, that none of these has quite addressed the issues from our perspective. In particular, we give a detailed discussion of BRST invariance in the presence of D-branes. In Appendix A we also illustrate the difference between our method of derivation of Schwarz's results for bound states of F- and D-strings, and that of Schmidhuber, by trying to derive the 11D membrane action from the type IIA membrane. The latter can be done only in the saddle-point approximation. This is the case for the passage from the IIB D-string to F-string as well that from the IIA D-membrane to 11D membrane actions in [14]. In our case the former is exact. In Appendix B we discuss the relation between the ten dimensional Born-Infeld action occurring in the work of CLNY [10] and the DBI action of Leigh [15].

2 The F-string action and SL(2,Z) duality in type IIB

In the discussion of singular string (or more generally p-brane) solutions of the low energy action [5], it is usually assumed that the singularity is supported by the explicit presence of a string. Thus, it is interpreted as the solution to the equations of motion coming from the original effective action plus the world sheet action of the string, in analogy with the particle case. However, as mentioned in the introduction, there is a

crucial difference between particle motions and that of the string. The statement that in string theory the background itself is a condensate of strings may be summarized by Tseytlin's equation [16] for the quantum string effective action.[3] This is a functional of the expectation value of the string field (whose low energy components are the metric, the dilaton, etc.) and is minimized with respect to it (and indeed may make sense only at the minimum),

$$\Gamma_{10} = \int_{\chi=2} e^{-S_2[X]} + \sum_{\chi=0,-2,\ldots} \int_{\chi} e^{-S_2}. \tag{2.1}$$

In this equation the functional integral is taken over the embedding functions X of the string world sheet in 10 dimensional space-time and intrinsic world sheet metrics, divided by the volume of 2D diffeomorphism and Weyl groups weighted by the world sheet σ-model action,

$$\begin{aligned} S_2 &= -T_2 \int d^2\sigma \left[\frac{1}{2}\sqrt{-\gamma}\gamma^{AB}\partial_A X^\mu \partial_B X^\nu g_{\mu\nu} e^{\phi/2} + \frac{1}{2!}\epsilon^{AB}\partial_A X^\mu \partial_B X^\nu B_{\mu\nu} \right. \\ &\qquad \left. - \frac{1}{4\pi T_2}\sqrt{-\gamma} R_{(2)}\phi \right], \end{aligned} \tag{2.2}$$

(with the target space metric being the canonical one). The sum is over the different world sheet topologies.[4] According to Tseytlin, the first term is, in fact, the classical effective action whose bosonic part (ignoring the RR sector) is given to leading order in α' by

$$I_{10} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left[R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2\cdot 3!} e^{-\phi} H^2 \right]. \tag{2.3}$$

The quantum equations of motion are then given by,

$$\frac{\delta\Gamma}{\delta\phi^i} = \frac{\delta I_{10}}{\delta\phi^i} - \sum_{\chi=0,-2,\ldots} \int_{\chi} e^{-S_2} \frac{\delta S_2}{\delta\phi^i} = 0. \tag{2.4}$$

In the above the ϕ_i represent the different background fields G, B, ϕ, etc. We will now argue that the terms coming from the addition of the 2D-string action to the 10D-effective

[3] This was used by Susskind and Uglum to provide support for their argument that black hole entropy can be understood in terms of strings [17].

[4] There is a subtlety involving the Möbius volume in the first term which we have ignored since it is irrelevant to our argument.

action in the work of [5] and in related subsequent work[5] are in fact obtained by approximating the leading string loop correction by its sigma model "classical" approximation.[6]

Just by using the condition that the string like solution that we are looking for preserves some supersymmetry one finds [5], [9];

$$ds^2 = A_2^{-3/4}(y)\eta_{\alpha\beta}dx^\alpha dx^\beta + A_2^{1/4}(y)\delta_{ij}dy^i dy^j, \tag{2.5}$$

$\alpha, \beta = 0, 1;\ i, j = 2, \ldots, 9$ and $e^{-\phi} = A_2^{1/2}(y),\ B_{01} = -e^{2\phi} = -A_2^{-1}(y)$. We want to evaluate the $\chi = 0$ term in (2.4) at its "classical" point. So, in addition to the above we put $\gamma_{AB} = \partial_A X^\mu \partial_B X^\nu g_{\mu\nu}$ and the ansatz $X^\alpha = \sigma^\alpha,\ \alpha = 0, 1;\quad X^i = \text{const.},\ i = 2, \ldots, 9$, which gives a classical solution. Substituting into the action we find $S_2 = 0$ so that the leading loop corrections in (2.4) are just what would come from adding the action S_2 to the effective action I_{10}.

Let us now review Schwarz's results [7]. In the bosonic sector of type IIB supergravity, there are a symmetric tensor $g_{\mu\nu}$, a dilaton ϕ, and a 2-form gauge potential $B^{(1)}_{\mu\nu}$ from the NS-NS sector, and a scalar field χ, another 2-form gauge potential $B^{(2)}_{\mu\nu}$, and a 4-form gauge potential $B_{\mu\nu\lambda\rho}$ from the R-R sector. The 5-form field strength associated with $B_{\mu\nu\lambda\rho}$ is self-dual, i.e., $F_5 = *F_5$ where $F_5 = dB_4$. It is impossible to write down an action when there is such a self-dual field strength. When $F_5 = 0$, however, one has (using the canonical metric) the type IIB supergravity action in a manifestly SL$(2, R)$-invariant form [6],

$$I_{10}^{\text{IIB}} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left[R + \frac{1}{4}\text{tr}(\partial\mathcal{M}\partial\mathcal{M}^{-1}) - \frac{1}{2\cdot 3!}\mathcal{H}^T\mathcal{M}\mathcal{H}\right]. \tag{2.6}$$

$\mathcal{M}$ is an SL$(2, R)$ matrix of the scalar fields,

$$\mathcal{M} = e^\phi \begin{pmatrix} |\lambda|^2 & \chi \\ \chi & 1 \end{pmatrix} \in \text{SL}(2, R). \tag{2.7}$$

λ is a complex scalar field defined by $\lambda = \chi + ie^{-\phi}$. $\mathcal{H}$ is a vector of the two 3-form field strengths:

$$\mathcal{H} = \begin{pmatrix} H^{(1)} \\ H^{(2)} \end{pmatrix}. \tag{2.8}$$

[5] See [9] and [18] for reviews.

[6] For related observations, see [19]. We thank A. A. Tseytlin for bringing this reference to our attention.

Under an SL$(2,R)$ transformation Λ,

$$\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,R), \tag{2.9}$$

$\mathcal{M} \to \Lambda \mathcal{M} \Lambda^T$ or, equivalently, $\lambda \to (a\lambda+b)/(c\lambda+d)$, and $\mathcal{H} \to (\Lambda^T)^{-1}\mathcal{H}$. This symmetry implies that the equations of motion yield a multiplet of (singular) string-like solutions carrying both NS-NS and R-R electric charges

$$\mathbf{q} e_2 = \frac{1}{\sqrt{2}\kappa} \int_{S^7} \mathcal{M}\, {}^*\mathcal{H}. \tag{2.10}$$

$e_2 = \sqrt{2}\kappa T_2$ is the charge of the fundamental string with tension T_2 and $\mathbf{q} = (q_1, q_2)^T$ is an SL$(2,Z)$ vector of integers in accordance with the usual Dirac argument. The tensions (in the canonical metric) are given by $T = T_2 \Delta_q^{1/2}$ and the second factor is the SL$(2,Z)$-invariant expression

$$\Delta_q = \mathbf{q}^T \mathcal{M}_0^{-1} \mathbf{q} = e^{\phi_0}(q_1 - q_2\chi_0)^2 + e^{-\phi_0} q_2^2. \tag{2.11}$$

The suffix "0" denotes the v.e.v. of each field. The quantization of charges implies that the symmetry of the system breaks down from SL$(2,R)$ to SL$(2,Z)$. The solutions are of course singular as is the case for the fundamental string. What one needs is a string action that supports both the NS-NS and the R-R charges. The obvious candidate is a string action (Nambu-Goto or Polyakov) with the tension T and the two-form couplings

$$-T_2 \int d^2\sigma \frac{1}{2} \epsilon^{AB} \partial_A X^\mu \partial_B X^\nu \mathcal{B}_{\mu\nu}^T \mathbf{q}. \tag{2.12}$$

In our earlier considerations we justified the addition of such terms for the NS-NS charge carrying (1,0) fundamental string as an approximate evaluation of loop effect. In the rest of the paper we will show how to obtain the same for strings with arbitrary (q_1, q_2) charged strings by what is essentially the same argument, but now extended to world sheets with boundaries coupled to D-branes (strings).

3 Tree-Level linearized equations of motion from BRST

In this section we discuss (with the modifications necessary for type IIB) the relevant parts of CLNY[10] (and [20]), which should be consulted for more details.[7]

The left-moving vertex operators are given by $V^{\mu}_{-1} = \psi^{\mu} e^{-\varphi} c e^{ik\cdot X_{\rm L}}$, $V^{b}_{-1} = 2\partial\xi e^{-2\varphi} c e^{ik\cdot X_{\rm L}}$, $V^{c}_{-1} = \frac{1}{2}\eta c e^{ik\cdot X_{\rm L}}$, along with fermion vertex operator, $V^{A}_{-1/2} = S^{A} e^{-\varphi/2} c e^{ik\cdot X_{\rm L}}$. (The subscript on V denotes the picture in which it is defined.) We will also need the operator $U^{B}_{\eta} = -2^{-3/2}\eta S^{B} e^{\varphi/2} c e^{ik\cdot X_{\rm L}}$ later on. In the above ψ^{μ} is the world sheet superpartner of X^{μ} and c, φ, η, ξ are ghost fields. The spin fields are defined by $S^{A} = e^{(A\cdot\rho)}$, where we have used the bosonization formula $\psi^{\pm j} = e^{\pm\rho_j}, j = 1, \ldots, 5$. A is a spinor weight of SO(10) with five components each of which takes $\pm 1/2$ with odd/even number of negative signs corresponding to chirality (dot/no dot).

By tensoring left-moving vertex operators given above with the corresponding right-moving ones, we get the σ-model interaction which describes type IIB superstring in the background of graviton h, dilaton Φ (which we have normalized differently from CLNY), antisymmetric gauge potential $B^{(1)}$ from the NS-NS sector, and three form field strength $H^{(2)}_3$ and one-form field strength H_1 derived from scalar field χ from the R-R sector:

$$\begin{aligned}\mathcal{L}_{\rm I} &= \frac{1}{2} h_{\mu\nu}(x) V^{\{\mu}_{-1}\tilde{V}^{\nu\}}_{-1} - \Phi(x)[V^{b}_{-1}\tilde{V}^{c}_{-1} - V^{c}_{-1}\tilde{V}^{b}_{-1}] + \frac{1}{2} B^{(1)}_{\mu\nu}(x) V^{[\mu}_{-1}\tilde{V}^{\nu]}_{-1} \\ &+ \frac{2^{-1/2}}{3!}[\not{H}^{(2)}(x) C]_{AB} V^{A}_{-1/2}\tilde{V}^{B}_{-1/2} + 2^{-1/2}[\not{H}_1(x) C]_{AB} V^{A}_{-1/2}\tilde{V}^{B}_{-1/2}. \qquad (3.1)\end{aligned}$$

In the expression above, the first three terms define $\mathcal{L}^{\rm NS}_{\rm I}$, the contribution from the NS-NS sector, and the rest define $\mathcal{L}^{\rm R}_{\rm I}$, that from the R-R sector. C is the spinor metric.

The BRST charge Q is given by $Q = Q_0 + Q_1 + Q_2$ where

$$Q_0 = \oint \frac{dz}{2\pi i} e^{\sigma} T, \quad Q_1 = \oint \frac{dz}{2\pi i}\left(\frac{1}{2} i\eta\psi\cdot\partial X e^{\varphi}\right), \quad Q_2 = \oint \frac{dz}{2\pi i}\left(-\frac{1}{4} e^{\sigma}\eta\partial\eta e^{2\varphi}\right). \qquad (3.2)$$

T is the total stress tensor, $T = -\frac{1}{2}\partial X\cdot\partial X + \frac{1}{2}\psi\cdot\partial\psi +$ ghosts. The action of the BRST operator on the left-moving vertex operators gives $[Q_0, V]_{\pm} = \frac{1}{2}k^2\partial c V$, $[Q_2, V]_{\pm} = 0$, $\{Q_1, V^{\mu}_{-1}\} = k_{\mu} V^{c}_{-1}$, $[Q_1, V^{c}_{-1}] = 0$, $[Q_1, V^{b}_{-1}] = k_{\mu} V^{\mu}_{-1}$, $[Q_1, V^{A}_{-1/2}] = U^{B}_{\eta}(\not{k})_{B}{}^{A}$. These

[7] In sections 3 and 4 we will work in a Euclidean target space. Also, in the rest of this paper we use $T_2 = 1/2\pi\alpha' = 1$.

lead to

$$[Q_0+\tilde{Q}_0,\mathcal{L}_{\rm I}]=\frac{1}{2}k^2\mathcal{L}_{\rm I}(\partial c+\bar{\partial}\tilde{c}), \tag{3.3}$$

$$[Q_1+\tilde{Q}_1,\mathcal{L}_{\rm I}^{\rm NS}]=-\frac{i}{2}(\partial^\nu(g_{\nu\mu}+B^{(1)}_{\nu\mu})+2\partial_\mu\Phi)[V^c_{-1}\tilde{V}^\mu_{-1}-V^\mu_{-1}\tilde{V}^c_{-1}], \tag{3.4}$$

$$\begin{aligned}[Q_1+\tilde{Q}_1,\mathcal{L}_{\rm I}^{\rm R}] \;&=\; 2^{-1/2}[\frac{1}{3!}\gamma^{\lambda\mu\nu\rho}\gamma^{11}k_\lambda H^{(2)}_{\mu\nu\rho}+\frac{1}{2}\gamma^{\nu\rho}k^\mu H^{(2)}_{\mu\nu\rho}+\gamma^{\mu\nu}\gamma^{11}k_\mu H_\nu+k^\mu H_\mu]_A{}^B\\ &\times(U^A_\eta\tilde{V}_{B,-1/2}+V^A_{-1/2}\tilde{U}^\eta_B). \end{aligned}\tag{3.5}$$

In deriving the last equation (which has been slightly modified from CLNY for future convenience), we used the chirality of the vertex operators $\gamma^{11}{}_A^BV_B=+V_A$, $\gamma^{11}{}_A^B\tilde{U}_B=-\tilde{U}_A$. Antisymmetrized products of gamma matrices are defined by $\gamma^{\mu_1\cdots\mu_n}=\frac{1}{n!}\sum_{\rm perms}\epsilon(p)\gamma^{\mu_{p(1)}}\cdots\gamma^{\mu_{p(n)}}$.

Linearized field equations are obtained from

$$(Q+\tilde{Q})\mathcal{L}_{\rm I}^{\rm NS}|\Omega\rangle=0. \tag{3.6}$$

$|\Omega\rangle$ is the SL_2-invariant vacuum. Thus, at tree level (linearized) we have, using (3.3):

$$\Box h_{\mu\nu}=0,\quad \Box B^{(1)}_{\mu\nu}=0,\quad \Box\Phi=0, \tag{3.7}$$

$$\Box H^{(2)}_{\mu\nu\lambda}=0,\quad \Box H_\mu=0. \tag{3.8}$$

The gauge condition for the graviton is obtained from eq.(3.4)

$$\partial^\nu(h_{\nu\mu}+B^{(1)}_{\nu\mu})+2\partial_\mu\Phi=0. \tag{3.9}$$

From eq.(3.5), we have

$$dH^{(2)}=0,\quad d\,{}^*H^{(2)}=0,\quad dH_1=0,\quad d\,{}^*H_1=0. \tag{3.10}$$

We can generalize the argument given in [10] to obtain (the massless sector of) the boundary state in the presence of nonzero constant gauge field strength F for D-branes (strings). This is essentially given by Li [11] but we need some details which are not

explicitly presented there. In NS-NS sector this state is given by

$$
\begin{aligned}
|B\rangle_{\mathrm{NS}} &= \int_{k_{\mathrm{L}},k_{\mathrm{R}}} \kappa[\det(1+\mathcal{F})]^{1/2}\bar{\mathcal{L}}_{\mathrm{I}}^{\mathrm{NS}}\frac{1}{2}(\partial c+\bar{\partial}\tilde{c})(0)|\Omega\rangle \\
&\equiv \int_{k_{\mathrm{L}},k_{\mathrm{R}}} |k_{\mathrm{L}},k_{\mathrm{R}};\mathrm{NS}\rangle \\
&\equiv \kappa\int_{k_{\mathrm{L}},k_{\mathrm{R}}} D_{\mathrm{NS}}(k_{\mathrm{L}},k_{\mathrm{R}})\frac{1}{2}(c_0+\tilde{c}_0)|\Omega\rangle
\end{aligned}
\tag{3.11}
$$

with the definition

$$
\bar{\mathcal{L}}_{\mathrm{I}}^{\mathrm{NS}} = V_{-1}^{T}(k_{\mathrm{L}})\underline{T}\tilde{V}_{-1}(k_{\mathrm{R}}) + [V_{-1}^{b}(k_{\mathrm{L}})\tilde{V}_{-1}^{c}(k_{\mathrm{R}}) - V_{-1}^{c}(k_{\mathrm{L}})\tilde{V}_{-1}^{b}(k_{\mathrm{R}})]. \tag{3.12}
$$

κ is the string coupling constant which later on we will set equal to the exponential of the dilaton. The definition of $\mathcal{F}$ and the O(10) rotation matrix $\underline{T}$ are given below. The integral over left- and right-moving momenta is defined by

$$
\int_{k_{\mathrm{L}},k_{\mathrm{R}}} \equiv \int d^{10}k_{\mathrm{L}}d^{10}k_{\mathrm{R}}\delta^{p+1}(k_{\|})\delta^{10}(k_{\mathrm{L}}-\underline{T}k_{\mathrm{R}}). \tag{3.13}
$$

The first δ-function restricts the solution to the momentum eigenstates with zero momentum in the parallel directions. The second δ-function constraint is explained below.

In the absence of gauge fields, we define $\underline{T}=T_0$ by

$$
T_0 = \mathrm{diag}[-\mathbf{1}_{p+1},\mathbf{1}_{9-p}], \tag{3.14}
$$

$\mathbf{1}_n$ is the $n\times n$ unit matrix. Equations (3.13) and (3.14) imply free Neumann boundary condition on $\alpha=0,\ldots,p$ and free Dirichlet boundary condition on $i=p+1,\ldots,9$ as is appropriate for a D-(p-)brane. Now, we turn on a gauge field coupled to the boundary with constant field strength $\mathcal{F}$. Taking into account the argument of [8], we introduce

$$
\mathcal{F} = F + B^{(1)}, \qquad \mathcal{F}^T = -\mathcal{F}. \tag{3.15}
$$

and write following [10], [11],

$$
\underline{T} = T(\mathcal{F}) = \frac{1-\mathcal{F}}{1+\mathcal{F}}T_0. \tag{3.16}
$$

Note that T is orthogonal because $\mathcal{F}$ is antisymmetric; $T^TT=TT^T=\mathbf{1}_{10}$. Now, we have the commutation relation

$$
[Q_1+\tilde{Q}_1,\bar{\mathcal{L}}_{\mathrm{I}}^{\mathrm{NS}}] = (k_{\mathrm{L}}^{\nu}T_{\nu\mu}-k_{\mu\mathrm{R}})V_{-1}^{c}(k_{\mathrm{L}})\tilde{V}_{-1}^{\mu}(k_{\mathrm{R}}) - (T_{\mu\nu}k_{\mathrm{R}}^{\nu}-k_{\mu\mathrm{L}})V_{-1}^{\mu}(k_{\mathrm{L}})\tilde{V}_{-1}^{c}(k_{\mathrm{R}}). \tag{3.17}
$$

BRST invariance requires that the r.h.s. of the equation above should vanish. Thus, we have

$$k_{\rm L}^{\nu}T_{\nu\mu}-k_{\mu{\rm R}}=0,\qquad T_{\mu\nu}k_{\rm R}^{\nu}-k_{\mu{\rm L}}=0. \tag{3.18}$$

This condition in fact requires that $\underline{T}$ is orthogonal. We see that the δ-function constraint on eq.(3.11) ensures $Q_1+\tilde{Q}_1$ invariance. Henceforth, we will be taking a static D-string so that we put

$$\mathcal{F}=\begin{pmatrix}\mathcal{F}_{\alpha\beta} & 0\\ 0 & 0\end{pmatrix},\qquad \alpha,\beta=0,1. \tag{3.19}$$

The boundary state in the R-R sector is expected to have the general form (in the $s_{\rm R}+s_{\rm L}=-1$ picture)

$$|B\rangle_{\rm R}=\int_{k_{\rm L},k_{\rm R}}\sum_{s}V_s^A L_{AB}\tilde{V}^B_{-1-s}\frac{c_0+\tilde{c}_0}{2}|\Omega\rangle, \tag{3.20}$$

where the sum is over half integers s. L_{AB} is to be determined by using space-time supersymmetry.

The boundary state is expected to be supersymmetric under a linear combination of the left and right supersymmetry (SUSY) generators

$$(\Lambda_r^A+\tilde{\Lambda}_r^B M_B{}^A(T))(|B\rangle_{\rm NS}+|B\rangle_{\rm R})=0, \tag{3.21}$$

where $M_B{}^A(T)$ is the spinor representation of O(10) rotation; i.e.,

$$T_{\mu\nu}\gamma^{\nu}_A{}^B=[M(T)^{-1}\gamma_\mu M(T)]_A{}^B. \tag{3.22}$$

In the case of D-string ($p=1$), one gets [10], [11]

$$[\not{k}_{\rm R}]_A{}^B=[M(T)^{-1}\not{k}_{\rm L}M(T)]_A{}^B, \tag{3.23}$$

$$M(T)=M\left(\frac{1-\mathcal{F}}{1+\mathcal{F}}\right)M(T_0)=[\det(1+\mathcal{F})]^{-1/2}e^{-\frac{1}{2}\not{\mathcal{F}}}(i\gamma_0\gamma_1). \tag{3.24}$$

The action of the SUSY generators on the vertex operators is given in a picture independent form by CLNY[10] (equation (3.37)). Using that and (3.21) one can determine [11] L_{AB}, and thus the R-R boundary state, which takes the form

$$|B\rangle_{\rm R}=\int_{k_{\rm L},k_{\rm R}}|k_{\rm L},k_{\rm R};{\rm R}\rangle, \tag{3.25}$$

$$
\begin{aligned}
|k_{\rm L},k_{\rm R};{\rm R}\rangle &= i\frac{\kappa}{\sqrt{2}}\sum_s V_s^A[\not{k}_{\rm L}e^{-\frac{1}{2}\not{\mathcal{F}}}(i\gamma_0\gamma_1)]_A{}^B\tilde{V}_{B,-1-s}\frac{c_0+\tilde{c}_0}{2}|\Omega\rangle \\
&\equiv \frac{\kappa}{\sqrt{2}}D_{\rm R}(k_{\rm L},k_{\rm R})\frac{c_0+\tilde{c}_0}{2}|\Omega\rangle. \qquad (3.26)
\end{aligned}
$$

This state is given as a sum over all the pictures which satisfies $s_{\rm R}+s_{\rm L}=-1$. For our calculations in the following sections, we choose the picture $s_{\rm R}=s_{\rm L}=-1/2$.[8]

4 Loop-Corrected Field Equations

In this section we calculate the loop-corrected string field equations in the presence of a D-string. We need to attach the boundary state obtained in the previous section to a sphere using the propagator [21], $\Pi\equiv\frac{1}{2}(b_0+\tilde{b}_0)(L_0+\tilde{L}_0)^{-1}$. So we define the states

$$
|D\rangle_{\rm NS}\equiv\Pi|B\rangle_{\rm NS}=\int_{k_{\rm L},k_{\rm R}}\frac{\kappa}{2k^2}D_{\rm NS}(k_{\rm L},k_{\rm R})|\Omega\rangle, \qquad (4.1)
$$

$$
|D\rangle_{\rm R}\equiv\Pi|B\rangle_{\rm R}=\int_{k_{\rm L},k_{\rm R}}\frac{\kappa}{\sqrt{2}}\frac{1}{2k^2}D_{\rm R}(k_{\rm L},k_{\rm R})|\Omega\rangle, \qquad (4.2)
$$

where we have used the fact that only the massless modes have been kept. The important point here is that even though the original boundary state is BRST invariant the state modified by the propagator is not. Thus, this BRST anomaly must be canceled by going offshell with the tree-level equations. The crucial point of [2] is that the modified field equations should be obtained from the condition, $(Q+\tilde{Q})|\Psi\rangle=0$, where the state is defined by adding eqs.(4.1) and (4.2) to the tree-level state

$$
|\Psi\rangle=(\mathcal{L}_{\rm I}^{\rm NS}+\mathcal{L}_{\rm I}^{\rm R})|\Omega\rangle+\kappa\int_{k_{\rm L},k_{\rm R}}\frac{1}{2k^2}\left(D_{\rm NS}+\frac{1}{\sqrt{2}}D_{\rm R}\right)|\Omega\rangle. \qquad (4.3)
$$

The condition, $(Q_0+\tilde{Q}_0)|\Psi\rangle=0$, gives the loop correction to the eqs.(3.7) and (3.8). In the NS-NS sector we obtain

$$
\Box h_{\mu\nu}=\kappa T_{\{\mu\nu\}}\delta^8(x_\perp)[\det(1+\mathcal{F})]^{1/2}, \qquad (4.4)
$$

$$
\Box B^{(1)}_{\mu\nu}=\kappa T_{[\mu,\nu]}\delta^8(x_\perp)[\det(1+\mathcal{F})]^{1/2}, \qquad (4.5)
$$

[8] For further discussion of this point see [10].

$$\Box \Phi = \kappa \delta^8(x_\perp)[\det(1+\mathcal{F})]^{1/2}, \tag{4.6}$$

and in the R-R sector

$$\Box H^{(2)}_{\mu\nu\lambda} = -\frac{\kappa}{2}\partial_{[\lambda} J_{\mu\nu]}, \tag{4.7}$$

$$\Box H_\mu = \frac{\kappa}{2}\mathcal{F}^{\lambda\sigma}\partial_\mu J_{\lambda\sigma}. \tag{4.8}$$

$x_\perp$ denotes the 8-directions transverse to the surface of D-string world sheet. J is a conserved 2-form current given by

$$J = \frac{1}{2}\int d^2\sigma \frac{1}{\sqrt{g}}\delta^{10}(f^\mu - x^\mu)\epsilon^{AB}\partial_A f^\lambda \partial_B f^\sigma g_{\lambda\mu} g_{\sigma\nu} dx^\mu \wedge dx^\nu. \tag{4.9}$$

We have introduced the D-brane embedding functions $f^\lambda(\sigma)$, σ being the D-sheet coordinates, and in our calculation we had specialized to flat space and the static gauge $f^\alpha = \delta^\alpha_A \sigma^A$, $f^i = 0$. The next condition

$$(Q_1 + \tilde{Q}_1)|\Psi\rangle = 0 \tag{4.10}$$

gives the loop correction to eqs. (3.10). No loop correction to NS-NS sector is given by this condition. The correction to R-R sector can be read off from

$$(Q_1 + \tilde{Q}_1)|D\rangle_{\mathrm{R}} = -\frac{\kappa}{2\sqrt{2}}\int_{k_{\mathrm{L}},k_{\mathrm{R}}} \left[U^A_\eta \tilde{V}_{B,-1/2} + V^A_{-1/2}\tilde{U}_{B,\eta}\right](\gamma^{\mu\nu} + \mathcal{F}^{\mu\nu})_A{}^B \frac{1}{2} J_{\mu\nu}|\Omega\rangle. \tag{4.11}$$

Thus, we get

$$dH^{(2)}_3 = 0, \quad dH_1 = 0, \quad \partial^\mu H^{(2)}_{\mu\nu\lambda} = i\frac{\kappa}{2}J_{\nu\lambda}, \quad \partial^\mu H_\mu = i\frac{\kappa}{4}\mathcal{F}^{\nu\lambda}J_{\nu\lambda}. \tag{4.12}$$

The last two equations in (4.12) can be written in form notation

$$d\,{}^*H^{(2)}_3 = i\frac{\kappa}{2}\,{}^*J, \qquad d\,{}^*H_1 = i\frac{\kappa}{2}\mathcal{F}\wedge {}^*J. \tag{4.13}$$

Now, recall that the current J satisfies the conservation equation $d\,{}^*J = 0$. The equations in (4.13) are consistent with it only when κ is constant. However, the coupling constant in string theory is actually $\kappa = e^\phi$, where the "curvature" dilaton ϕ is related to the "ghost" dilaton Φ by

$$\phi(x) = \Phi(x) + \frac{1}{4}h^\mu{}_\mu(x). \tag{4.14}$$

Thus, as pointed out in [10], we need to modify eq.(4.15) to get

$$d^*\Big(e^{-\phi}H_3^{(2)}\Big)=\frac{i}{2}{}^*J,\qquad d^*\Big(e^{-\phi}H_1\Big)=\frac{i}{2}\mathcal{F}\wedge{}^*J. \tag{4.15}$$

These modifications may be justified from our BRST point of view by including the extra contribution due to a linear dilaton in the BRST charge (see also [13], [22]). When there is a linear dilaton $\phi=X^0$ in the background the stress tensor is

$$T_B=-\frac{1}{2}\partial X\cdot\partial X+\frac{1}{2}\psi\cdot\partial\psi+\partial^2X^0. \tag{4.16}$$

It forms a supermultiplet with the superconformal current;

$$T_F=-\frac{1}{2}\psi\cdot\partial X+\partial\psi^0. \tag{4.17}$$

The second term on the r.h.s. modifies Q_1 in equation (3.2). The commutators containing Q_1 thus get extra terms

$$[Q_1,V^A_{-1/2}]=U^B_\eta\left\{(\not k)_B{}^A+i(\gamma^0)_B{}^A\right\}, \tag{4.18}$$

and

$$\begin{aligned}
[Q_1+\tilde{Q}_1,\mathcal{L}^{\rm R}_{\rm I}] &= 2^{-1/2}\Big[\frac{1}{3!}\gamma^{\lambda\mu\nu\rho}\gamma^{11}k_\lambda H^{(2)}_{\mu\nu\rho}+\frac{1}{2}\gamma^{\nu\rho}k^\mu H^{(2)}_{\mu\nu\rho}+\gamma^{\mu\nu}\gamma^{11}k_\mu H_\nu+k^\mu H_\mu\\
&\quad+i\left\{\frac{1}{3!}\gamma^{0\mu\nu\rho}\gamma^{11}H^{(2)}_{\mu\nu\rho}+\frac{1}{2}\gamma^{\nu\rho}\delta^{0\mu}H^{(2)}{}_{\mu\nu\rho}+\gamma^{0\nu}\gamma^{11}H_\nu+\delta^{0\mu}H_\mu\right\}\Big]_A{}^B\\
&\quad\times(U^A_\eta\tilde{V}_{B,-1/2}+V^A_{-1/2}\tilde{U}^\eta_B)\\
&= 2^{-1/2}(-i)\Big[\frac{1}{3!}\gamma^{\lambda\mu\nu\rho}\gamma^{11}(\partial_\lambda-\partial_\lambda\phi)H^{(2)}_{\mu\nu\rho}+\frac{1}{2}\gamma^{\nu\rho}(\partial^\mu-\partial^\mu\phi)H^{(2)}_{\mu\nu\rho}\\
&\quad+\gamma^{\mu\nu}\gamma^{11}(\partial_\mu-\partial_\mu\phi)H_\nu+(\partial^\mu-\partial^\mu\phi)H_\mu\Big]_A{}^B\\
&\quad\times(U^A_\eta\tilde{V}_{B,-1/2}+V^A_{-1/2}\tilde{U}^\eta_B)\\
&= -2^{-1/2}ie^\phi\Big[\frac{1}{3!}\gamma^{\lambda\mu\nu\rho}\gamma^{11}\partial_\lambda\left(e^{-\phi}H^{(2)}_{\mu\nu\rho}\right)+\frac{1}{2}\gamma^{\nu\rho}\partial^\mu\left(e^{-\phi}H^{(2)}_{\mu\nu\rho}\right)\\
&\quad+\gamma^{\mu\nu}\gamma^{11}\partial_\mu\Big(e^{-\phi}H_\nu\Big)+\partial^\mu\Big(e^{-\phi}H_\mu\Big)\Big]_A{}^B(U^A_\eta\tilde{V}_{B,-1/2}+V^A_{-1/2}\tilde{U}^\eta_B).
\end{aligned} \tag{4.19}$$

We used $k_\mu=-i\partial_\mu$, and $\delta^0_\mu=\partial_\mu X^0=\partial_\mu\phi$. Then from (4.10) we have the modified eq.(4.15) and similarly modified Bianchi identities.

It is natural to define

$$\tilde{H}_3^{(2)} \equiv e^{-\phi} H_3^{(2)}, \qquad \tilde{H}_1 \equiv e^{-\phi} H_1. \tag{4.20}$$

The equations (4.15) are now written as

$$d\,{}^*\tilde{H}_3^{(2)} = \frac{i}{2}\,{}^*J, \qquad d\,{}^*\tilde{H}_1 = \frac{i}{2}\mathcal{F} \wedge {}^*J, \tag{4.21}$$

and the Bianchi identities become

$$d\tilde{H}_3^{(2)} = 0, \qquad d\tilde{H}_1 = 0. \tag{4.22}$$

The effective action which gives these equations of motion and Bianchi identities should be in the form

$$\begin{aligned} S &\sim \frac{1}{2}\int_{M_{10}} \left(\tilde{H}_3^{(2)} \wedge {}^*\tilde{H}_3^{(2)} + \tilde{H}_1 \wedge {}^*\tilde{H}_1 + i\,{}^*J \wedge B^{(2)} + i\chi\,{}^*J \wedge \mathcal{F}\right) \\ &= \frac{1}{2}\int_{M_{10}} \left(\tilde{H}_3^{(2)} \wedge {}^*\tilde{H}_3^{(2)} + \tilde{H}_1 \wedge {}^*\tilde{H}_1\right) + \frac{i}{2}\int_{\text{D}-\text{sheet}} \left(\tilde{B}^{(2)} + \chi\tilde{\mathcal{F}}\right), \end{aligned} \tag{4.23}$$

with $\tilde{H}_3^{(2)} = dB^{(2)}$, $\tilde{H}_1 = d\chi$. $\tilde{B}^{(2)}$ and $\tilde{\mathcal{F}}$ are the pullbacks of $B^{(2)}$ and $\mathcal{F}$ to the D-sheet. Note that the redefined R-R fields appear without a dilaton factor.

5 From D-string action to F-string action

The action which reproduces the right hand sides of the loop-corrected equations is obtained by adding the DBI action given by Leigh [15] and the last two terms in eq.(4.23);

$$S_D = \int_{\text{D}-\text{sheet}} \left[e^{-\phi}\sqrt{\det(\tilde{g} + \tilde{\mathcal{F}})} + i\tilde{B}^{(2)} + i\chi\tilde{\mathcal{F}}\right]. \tag{5.1}$$

$\tilde{g}$ is the pullback of g to the D-sheet. The only non-trivial equation of motion to check is that for the graviton since, in particular, it is not obvious how the transverse contributions to the right hand side of (4.4) arise. The point to note here is that the variation must be performed keeping Φ rather than ϕ fixed. Putting $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$, using (4.14), and

going to the static gauge, we may write the metric-dependent part of (5.1) as

$$
\begin{aligned}
S_D &\sim \int d^{10}x\delta^8(x_\perp)e^{-\Phi-\frac{1}{4}h^\mu_\mu}\sqrt{\det{}_{\|}[1+h+\mathcal{F}]} \\
&= \int d^{10}x\delta^8(x_\perp)e^{-\Phi}\sqrt{\det{}_{\|}[1+\mathcal{F}]}\left[1+\frac{1}{4}\mathrm{tr}_{\|}(\frac{1-\mathcal{F}}{1+\mathcal{F}}h)-\frac{1}{4}\mathrm{tr}_\perp h\right].
\end{aligned} \tag{5.2}
$$

It is worth noting that the sign change in the transverse directions arises from the dilaton term where the transition from ϕ to Φ is the usual T-duality measure transformation.

We are now ready to show how this D-string action becomes a fundamental string action with the tension given by Schwarz's formula. We need to start with the action for q_2 D-strings so we take the following world sheet Lagrangian[9]

$$
\begin{aligned}
\mathcal{L} &= q_2\left\{e^{-\phi}\sqrt{-\det(\tilde{g}+\tilde{\mathcal{F}})}+\frac{1}{2}\chi\epsilon^{\alpha\beta}\tilde{\mathcal{F}}_{\alpha\beta}+\frac{1}{2}\epsilon^{\alpha\beta}\tilde{B}^{(2)}_{\alpha\beta}\right\} \\
&= q_2e^{-\phi}\sqrt{-\det(\tilde{g}+\tilde{\mathcal{F}})}+q_2\chi\tilde{\mathcal{F}}_{01}+q_2\tilde{B}^{(2)}_{01}.
\end{aligned} \tag{5.3}
$$

Here, $\tilde{\mathcal{F}}_{01}=\dot{A}_1-\partial_1A_0-\tilde{B}^{(1)}_{01}$. We can rewrite the DBI action (first term)

$$
\begin{aligned}
\sqrt{-\det(\tilde{g}+\tilde{\mathcal{F}})} &= \sqrt{-\det\tilde{g}}\left[1-\frac{1}{2}\mathrm{tr}(\tilde{g}^{-1}\tilde{\mathcal{F}}\tilde{g}^{-1}\tilde{\mathcal{F}})\right]^{1/2} \\
&= \sqrt{-\det\tilde{g}}\sqrt{1+(\tilde{\mathcal{F}}_{01})^2(\det\tilde{g})^{-1}} \\
&= \sqrt{-\det\tilde{g}-(\tilde{\mathcal{F}}_{01})^2}.
\end{aligned} \tag{5.4}
$$

The momentum conjugate to the gauge potential A is

$$
\pi_1=-\frac{q_2e^{-\phi}\tilde{\mathcal{F}}_{01}}{\sqrt{-\det\tilde{g}-(\tilde{\mathcal{F}}_{01})^2}}+q_2\chi, \qquad \pi_0=0, \tag{5.5}
$$

which gives the Hamiltonian

$$
H=-\sqrt{-\det\tilde{g}}\sqrt{q_2^2e^{-2\phi}+(\pi_1-q_2\chi)^2}-A_0\partial_1\pi_1+\partial_1(\pi_1A_0)+\pi_1\tilde{B}^{(1)}_{01}+q_2\tilde{B}^{(2)}_{01}. \tag{5.6}
$$

[9]Note that from now on we will be working in a Minkowskian signature metric space-time.

We now choose to define the theory using the Hamiltonian form of the path integral.[10] After canceling the gauge group volume against $\int d\pi_0$, we have

$$\begin{aligned} Z &= \int [d\pi_1][dA_0][dA_1] \exp\left[i\int d^2\sigma \left\{\pi_1 \dot{A}_1 - H(A,\pi_1)\right\}\right] \\ &= \int [d\pi_1][dA_0][dA_1] \exp\Big[i\int d^2\sigma \Big\{-A_1\dot{\pi}_1 + A_0\partial_1\pi_1 \\ &\quad + \sqrt{-\det\tilde{g}}\sqrt{q_2^2 e^{-2\phi} + (\pi_1 - q_2\chi)^2} - \pi_1\tilde{B}_{01}^{(1)} - q_2\tilde{B}_{01}^{(2)}\Big\}\Big]. \end{aligned} \tag{5.7}$$

We can carry out the integrals over A_0 and A_1 to give δ-functions

$$\begin{aligned} Z &= \int [d\pi_1]\delta(\dot{\pi}_1)\delta(\partial_1\pi_1) \\ &\quad \times \exp\Big[i\int d^2\sigma\Big\{\sqrt{-\det\tilde{g}}\sqrt{q_2^2 e^{-2\phi} + (\pi_1 - q_2\chi)^2} + \pi_1\tilde{B}_{01}^{(1)} + q_2\tilde{B}_{01}^{(2)}\Big\}\Big]. \end{aligned} \tag{5.8}$$

Because of the δ-functions, the integral reduces to the one over the zero-mode of π_1. The zero mode is quantized when x^1 is compactified on a circle [8]. Thus the integral is replaced by a sum;

$$Z = \sum_{q_1} \exp\left[i\int d^2\sigma\left\{\sqrt{-\det\tilde{g}}\sqrt{q_2^2 e^{-2\phi} + (q_1 - q_2\chi)^2} + q_1\tilde{B}_{01}^{(1)} + q_2\tilde{B}_{01}^{(2)}\right\}\right]. \tag{5.9}$$

The combination $q_1\tilde{B}_{01}^{(1)} + q_2\tilde{B}_{01}^{(2)}$ is what we saw in eq.(2.12). We can read off the string tension

$$T = \sqrt{\frac{q_2^2}{\kappa^2} + (q_1 - q_2\chi)^2}. \tag{5.10}$$

In the canonical metric this expression is in the form $T \sim \Delta_q^{1/2}$ given in section 2. These facts support the SL$(2, Z)$ invariance of the theory.

6 Discussion

This action now provides the support for the singularity in the solutions generated by Schwarz [7]. There are two issues remaining to be discussed. One is the fact that Leigh's

[10] This is of course not an unambiguous choice, since one may also define directly a Lagrangian path integral. However, our choice is the one which is directly related to the operator formulation and hence also to the argument of [8].

action leads to string actions in the Nambu-Goto rather than the Polyakov forms. This is easily understood from our earlier considerations that the string actions arise in this form from the classical σ-model approximation to the one-loop string term. The other is the fact that string theory calculations appear to be yielding objects which have space-time singularities. We believe that the resolution of this puzzle comes from the following consideration. The string like solutions coming just from I_{10} are such that $\kappa = e^{\phi}$ is zero on the singularity. Thus, the string action is relevant only for the equations that relate the charge to the string tension namely (4.15) which does not have a vanishing coupling constant factor on the right hand side. This raises the question whether space-time singularities have physical reality. We should also mention here reference [23] where it is pointed out that the singularity becomes invisible to strings when the level-matching conditions are satisfied.[11]

We believe that we have shed some light in this paper on the relation between the first quantized (σ- model) string and its counter part which occurs as a solution to effective low energy field equations. Polchinski's observations [4] hold out the promise that $p > 1$ branes may also be analyzed in terms of world sheet considerations thus possibly obviating the necessity for the quantization of $p > 1$ actions. Perhaps some light on M theory may also be shed by such considerations.

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Appendix A: Membrane Action

In ref.[14], Nambu-Goto-type actions are obtained starting from DBI actions for both strings and membranes. This derivation relies on the saddle-point approximation. As we saw in section 5, in the case of strings, we can obtain the same result without making

[11] It should be noted that for a boundary state the BRST condition plus the boundary conditions on the ghosts imply the level matching conditions $L_n - \tilde{L}_{-n} = 0$ as well as $F_n - \tilde{F}_{-n} = 0$.

any approximation. In this appendix, we apply the method used to derive eq.(5.9) to the case of membrane. It turns out that in this case we are unable to obtain the final result without using a saddle point approximation, unlike in the case of the string. We start with the effective type IIA D-membrane action

$$\int d^3\sigma \left\{ e^{-\phi}\sqrt{-\det(g+\mathcal{F})} + \frac{1}{6}\epsilon^{\alpha\beta\gamma}A_{\alpha\beta\gamma} - \frac{1}{2}\epsilon^{\alpha\beta\gamma}C_\alpha\mathcal{F}_{\beta\gamma} \right\}. \tag{A.1}$$

Here, $\alpha, \beta, \gamma = 0, 1, 2$. For simplicity, we consider the case where the space-time metric is Minkowskian. Then, we have

$$\begin{aligned}\sqrt{-\det(\eta+\mathcal{F})} &= \left[1 - \frac{1}{2}\mathrm{tr}(\eta^{-1}\mathcal{F}\eta^{-1}\mathcal{F})\right]^{1/2} \\ &= \sqrt{1 - \mathcal{F}_{01}^2 - \mathcal{F}_{02}^2 + \mathcal{F}_{12}^2}.\end{aligned} \tag{A.2}$$

The canonical momenta are given by

$$\pi_0 = 0, \tag{A.3}$$

$$\pi_1 = -e^{-\phi}\mathcal{F}_{01}[1 - \mathcal{F}_{01}^2 - \mathcal{F}_{02}^2 + \mathcal{F}_{12}^2]^{-1/2} - C_2, \tag{A.4}$$

$$\pi_2 = -e^{-\phi}\mathcal{F}_{02}[1 - \mathcal{F}_{01}^2 - \mathcal{F}_{02}^2 + \mathcal{F}_{12}^2]^{-1/2} + C_1, \tag{A.5}$$

and the Hamiltonian is

$$\begin{aligned}H = &- \sqrt{1+\mathcal{F}_{12}^2}\sqrt{e^{-2\phi} + (\pi_1 + C_2)^2 + (\pi_2 - C_1)^2} + C_0\mathcal{F}_{12} + \pi_1\partial_1 A_0 + \pi_2\partial_2 A_0 \\ &- \frac{1}{6}\epsilon^{\alpha\beta\gamma}A_{\alpha\beta\gamma} + \pi_1 B_{01} + \pi_2 B_{02}.\end{aligned} \tag{A.6}$$

Although we have gotten rid of the electric fields, the magnetic field $\mathcal{F}_{12}$ remains in the Hamiltonian; this is the main difference from the D-string case. The path integral goes almost the same way as that in the string case (again the integral over π_0 is cancelled by the group volume):

$$\begin{aligned}Z &= \int [d\pi_1][d\pi_2][dA_0][dA_1][dA_2]\exp\left[i\int d^3\sigma\{\pi_1\dot{A}_1 + \pi_2\dot{A}_2 - H\}\right] \\ &= \int [d\pi_1][d\pi_2][dA_0][dA_1][dA_2] \\ &\quad \times \exp\left[i\int d^3\sigma\Big\{(\partial_1\pi_1 + \partial_2\pi_2)A_0 + \pi_1\partial_0 A_1 + \pi_2\partial_0 A_2 - C_0\mathcal{F}_{12}\right.\end{aligned}$$

$$
\begin{aligned}
&+\sqrt{1+\mathcal{F}_{12}^2}\sqrt{e^{-2\phi}+(\pi_1+C_2)^2+(\pi_2-C_1)^2} \\
&+\frac{1}{6}\epsilon^{\alpha\beta\gamma}A_{\alpha\beta\gamma}-\pi_1 B_{01}-\pi_2 B_{02}\Big\}\Big] \\
= &\int[d\pi_1][d\pi_2][dA_1][dA_2]\delta(\partial_1\pi_1+\partial_2\pi_2) \\
&\times\exp\Big[i\int d^3\sigma\Big\{\pi_1\partial_0 A_1+\pi_2\partial_0 A_2-C_0\mathcal{F}_{12} \\
&+\sqrt{1+\mathcal{F}_{12}^2}\sqrt{e^{-2\phi}+(\pi_1+C_2)^2+(\pi_2-C_1)^2} \\
&+\frac{1}{6}\epsilon^{\alpha\beta\gamma}A_{\alpha\beta\gamma}-\pi_1 B_{01}-\pi_2 B_{02}\Big\}\Big].
\end{aligned}
\tag{A.7}
$$

δ-function gives $\pi_1=-\partial_2 y,\ \pi_2=\partial_1 y$ for a scalar function y:

$$
\begin{aligned}
Z \sim &\int[dA_1][dA_2]\exp\Big[i\int d^3\sigma\Big\{\sqrt{1+\mathcal{F}_{12}^2}\sqrt{e^{-2\phi}+(\partial_1 y-C_1)^2+(\partial_2 y-C_2)^2} \\
&+(\partial_0 y-C_0)\mathcal{F}_{12}+\frac{1}{6}\epsilon^{\alpha\beta\gamma}(A_{\alpha\beta\gamma}+3\partial_\alpha y B_{\beta\gamma})\Big\}\Big].
\end{aligned}
\tag{A.8}
$$

It is impossible to integrate exactly over the magnetic field. Taking a variation of the action with respect to $\mathcal{F}_{12}$ to find a saddle point, we get

$$
\mathcal{F}_{12}=-\frac{\partial_0 y-C_0}{e^{-2\phi}+\eta^{\alpha\beta}(\partial_\alpha y-C_\alpha)(\partial_\beta y-C_\beta)}.
\tag{A.9}
$$

$\mathcal{F}_{12}$ can be eliminated in the action to get

$$
\int d^3\sigma\Big\{\sqrt{e^{-2\phi}+\eta^{\alpha\beta}(\partial_\alpha y-C_\alpha)(\partial_\beta y-C_\beta)}+\frac{1}{6}\epsilon^{\alpha\beta\gamma}(A_{\alpha\beta\gamma}+3\partial_\alpha y B_{\beta\gamma})\Big\}.
\tag{A.10}
$$

This result can be generalized to general metric $g_{\alpha\beta}$ and we recover Schmidhuber's derivation of the bosonic part of 11dimensional supermembrane action [24].

$$
Z=\exp\left[i\int\left\{\sqrt{-\det\hat{g}}+\frac{1}{6}\epsilon^{\alpha\beta\gamma}\hat{A}_{\alpha\beta\gamma}\right\}\right],
\tag{A.11}
$$

where

$$
\hat{g}_{\alpha\beta}=g_{\alpha\beta}e^{-2\phi/3}+e^{4\phi/3}(\partial_\alpha y-C_\alpha)(\partial_\beta y-C_\beta),
\tag{A.12}
$$

$$
\hat{A}_{\alpha\beta\gamma}=A_{\alpha\beta\gamma}+3\partial_\alpha y B_{\beta\gamma}.
\tag{A.13}
$$

Appendix B: The Born-Infeld action and Leigh's DBI action

In this appendix we would like to discuss the relation between the Born-Infeld action that comes as a prefactor in the CLNY [10] construction of the NS-NS part of the boundary state equation (3.11) and the DBI action of Leigh [15]. In the body of the text we considered a static D-string so that the gauge field takes the form (3.19) and then the relation is trivial in the sense that the determinant of the ten-dimensional matrix $1+\mathcal{F}$ is equal to the determinant of the two-dimensional matrix that comes in Leigh's action in flat space and in static gauge. We will now show this equivalence in the case that is still restricted to flat space but now allowing for general motions of the D-string.

$$[\mathcal{F}]_{\mu\nu} = \begin{pmatrix} \mathcal{F}_{\alpha\beta} & \mathcal{F}_{\alpha j} \\ \mathcal{F}_{i\beta} & \mathcal{F}_{ij} \end{pmatrix} = \begin{pmatrix} \mathcal{F}_{\alpha\beta} & \partial_\alpha A_j \\ -\partial_\beta A_i & 0 \end{pmatrix} = \begin{pmatrix} \mathcal{F} & Y \\ -Y^T & 0 \end{pmatrix}, \tag{B.1}$$

where we defined

$$Y_{\alpha j} \equiv \mathcal{F}_{\alpha j}. \tag{B.2}$$

Recall that α and β are coordinates tangential to the D-p-brane world volume, while i and j are transverse to it, i.e. $\alpha, \beta = 0, \ldots, p$, and $i, j = p+1, \ldots, 9$. In the static gauge

$$f^\alpha = \delta^\alpha_A \sigma^A, \tag{B.3}$$

where σ^A is the p-brane world volume coordinate. Along with the choice of gauge potential,

$$A_i = f^i(\sigma^A) = f^i(X^\alpha), \tag{B.4}$$

the (flat space) DBI action is written as

$$\det(\tilde{g} + \tilde{\mathcal{F}}) = \det(1 + \mathcal{F} + YY^T). \tag{B.5}$$

We expand the D-brane action up to fourth order in A and Y

$$\begin{aligned} \ln\det(1+\mathcal{F}+YY^T) &= \frac{1}{2}\ln\det(1+\mathcal{F}+YY^T)(1-\mathcal{F}+YY^T) \\ &= \frac{1}{2}\mathrm{tr}\left(2YY^T - \mathcal{F}^2 - (YY^T)^2 - \frac{1}{2}\mathcal{F}^4 + 2\mathcal{F}^2YY^T\right). \end{aligned} \tag{B.6}$$

The Born-Infeld action of CLNY up to the same order is

$$\ln\det\begin{pmatrix} 1+\mathcal{F} & Y \\ -Y^T & 1 \end{pmatrix} = \frac{1}{2}\ln\det\left[\begin{pmatrix} 1+\mathcal{F} & Y \\ -Y^T & 1 \end{pmatrix}\begin{pmatrix} 1-\mathcal{F} & -Y \\ Y^T & 1 \end{pmatrix}\right]$$

$$
\begin{aligned}
&= \frac{1}{2}\ln\det\left\{1+\begin{pmatrix} YY^T-\mathcal{F}^2 & -\mathcal{F}Y \\ Y^T\mathcal{F} & Y^TY \end{pmatrix}\right\} \\
&= \frac{1}{2}\mathrm{tr}\left\{2YY^T-\mathcal{F}^2-(YY^T)^2-\frac{1}{2}\mathcal{F}^4+2\mathcal{F}^2YY^T\right\}. \quad \text{(B.7)}
\end{aligned}
$$

Equations.(B.6) and (B.7) show that the DBI action of Leigh and the Born-Infeld action of CLNY are equivalent up to fourth order in A and Y. Since in the $p=1$ case both actions are of this order this is sufficient to establish the equivalence for this case. These two actions are surely equivalent for any p-brane but we have not tried to establish this in general.

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